

Online Supplemental Appendix to New Methods for Mixed-Frequency Modeling Using Machine Learning: With An Application to Forecasting Stock Returns

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Abstract

This supplemental appendix provides additional empirical results supporting the analysis presented in the main paper. To improve the readability of the manuscript, the main text focuses on the forecasting results for SPDR S&P 500 ETF (SPY), while corresponding results for the NVIDIA (NVDA), JPMorgan Chase (JPM), Eli Lilly (LLY), and Exxon Mobil (XOM) are reported here. For each asset, we present mean squared forecast error (MSFE) comparisons, Diebold–Mariano (DM) tests, and Model Confidence Set (MCS) results.

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1. Additional Empirical Results

This section reports supplementary forecasting results for NVDA, JPM, LLY, and XOM. For each asset, we present mean squared forecast error (MSFE) comparisons, Diebold–Mariano (DM) test results, and Model Confidence Set (MCS) results. These tables complement the SPY results reported in the main text and allow readers to assess the robustness of the proposed methods across different assets and sectors.

Table 1: MSFEs of “Learner Models” Using Different Mixed-Frequency Methods

NVDA Monthly Returns

Learner Model	Mixed-Frequency Method			
	Monthly	HPE	GHPA	DMHTA
Sample Period: 2017/12/01 – 2023/12/01				
Random Walk	2.374	–	–	2.364**
AR Model	2.329	–	–	2.328
RanForest	2.369	2.326	2.290	2.314
Boosting	2.570	2.475	2.314	2.371
SVM-RBF	3.622	2.276**	2.241**	2.289***
SVM-Poly	2.336	2.337	2.319	2.276
LASSO	2.315	3.231	2.192	2.281
Ridge	3.113	5.970	2.150*	2.206**
Nnet2	4.412	2.328**	2.303**	2.360***
Nnet3	2.882	2.361	2.203	2.362**
LSTM	2.534	2.374	2.172	2.335
Sample Period: 2017/12/01 – 2020/02/29				
Random Walk	1.740	–	–	1.737
AR Model	1.718	–	–	1.718
RanForest	1.744	1.698	1.713	1.764
Boosting	2.077	1.564	1.675	1.758
SVM-RBF	2.672	1.773	1.670*	1.742**
SVM-Poly	1.703	1.728	1.732	1.787
LASSO	1.789	1.504	1.640	1.730
Ridge	1.765	1.906	1.623	1.723
Nnet2	2.572	1.707	1.676	1.741**
Nnet3	1.732	1.747	1.626	1.741
LSTM	1.857	1.847	1.662	1.732
Sample Period: 2020/02/29 – 2023/12/01				
Random Walk	2.755	–	–	2.740**
AR Model	2.696	–	–	2.694
RanForest	2.744	2.703	2.636	2.644
Boosting	2.866	3.022	2.697	2.739
SVM-RBF	4.192	2.578*	2.583	2.617***
SVM-Poly	2.715	2.703	2.671	2.569
LASSO	2.631	4.268	2.524	2.612
Ridge	3.921	8.408	2.467	2.497**
Nnet2	5.516	2.700**	2.679*	2.732***
Nnet3	3.573	2.729	2.549	2.734**
LSTM	2.940	2.691	2.478	2.697

Notes: All reported MSFE entries are actual MSFEs multiplied by 10^{-2} . Bold values indicate the lowest MSFE within each model row. Superscripts denote statistical significance based on the application of one-sided Harvey–Leybourne–Newbold corrected Diebold–Mariano tests comparing each high-frequency method with the Monthly benchmark, where starred entries correspond to the following p-values: $*p < 0.10$, $**p < 0.05$, $***p < 0.01$. Monthly refers to the benchmark mixed-frequency method that uses solely monthly-frequency predictors. DMHTA denotes Diagonal Multi-Horizon Temporal Aggregation. HPE denotes High-Frequency Path Embedding. GHPA denotes Group-Structured High-Frequency Path Embedding with Autoencoding.

Table 2: Model Confidence Set p -Values*NVDA Monthly Returns*

Learner Model	Mixed-Frequency Method			
	Monthly	HPE	GHPA	DMHTA
Sample Period: 2017/12/01 – 2023/12/01				
Random Walk	0.0830	–	–	1.0000
AR Model	0.4006	–	–	1.0000
RanForest	0.8444	0.8866	1.0000	0.8866
Boosting	0.6110	0.6110	1.0000	0.8822
SVM-RBF	0.0022	1.0000	0.8486	0.9778
SVM-Poly	0.9594	0.9594	0.9594	1.0000
LASSO	0.9426	0.3652	0.9426	1.0000
Ridge	0.1442	0.0452	0.7708	1.0000
Nnet2	0.1006	1.0000	0.4450	0.8652
Nnet3	0.4588	1.0000	0.8708	0.9980
LSTM	0.7512	0.7264	1.0000	0.7696
Sample Period: 2017/12/01 – 2020/02/29				
Random Walk	0.6460	–	–	1.0000
AR Model	0.9296	–	–	1.0000
RanForest	0.9364	1.0000	0.5092	0.9364
Boosting	0.3136	1.0000	0.1228	0.5596
SVM-RBF	0.2034	0.7542	1.0000	0.8194
SVM-Poly	0.9978	1.0000	0.9978	0.7956
LASSO	0.8214	1.0000	0.6336	0.8214
Ridge	0.8548	0.4212	1.0000	0.8548
Nnet2	0.2264	1.0000	0.3276	0.7168
Nnet3	0.9906	1.0000	0.9546	0.9906
LSTM	0.4572	0.4232	1.0000	0.7866
Sample Period: 2020/02/29 – 2023/12/01				
Random Walk	0.0760	–	–	1.0000
AR Model	0.3692	–	–	1.0000
RanForest	0.7910	0.7910	1.0000	0.8884
Boosting	0.8830	0.0550	1.0000	0.8830
SVM-RBF	0.0100	1.0000	0.5992	0.9588
SVM-Poly	0.9280	0.9280	0.9280	1.0000
LASSO	0.9456	0.2160	0.9456	1.0000
Ridge	0.1054	0.0140	0.6794	1.0000
Nnet2	0.1218	0.9950	0.6720	1.0000
Nnet3	0.3286	0.9640	0.9122	1.0000
LSTM	0.9492	0.9492	1.0000	0.9492

Notes: Each cell reports the MCS p -value from the range statistic (T_R) of Hansen et al. (2011). For each model row, the MCS is computed *across* the four prediction frameworks: Monthly, HPE, GHPA, and DMHTA. A p -value of 1.0000 identifies the best-performing framework for that model; a value close to 0 indicates the framework eliminated first (worst). Bold values denote frameworks with p -value > 0.25 , indicating strong inclusion in the MCS. A framework is excluded from the $\alpha = 10\%$ MCS if its p -value < 0.10 .

Table 3: MSFEs of “Learner Models” Using Different Mixed-Frequency Methods

JPM Monthly Returns

Learner Model	Mixed-Frequency Method			
	Monthly	HPE	GHPA	DMHTA
Sample Period: 2017/12/01 – 2023/12/01				
Random Walk	0.616	–	–	0.616
AR Model	0.621	–	–	0.620
RanForest	0.644	0.633	0.606	0.634
Boosting	0.716	0.672	0.643	0.617*
SVM-RBF	1.799	0.620***	0.626***	0.706***
SVM-Poly	1.915	0.796***	0.832***	1.613
LASSO	0.616	0.616	0.617	0.622
Ridge	1.258	0.622**	0.617**	0.624***
Nnet2	1.387	0.673**	0.901*	0.617***
Nnet3	0.837	0.633	0.646	0.617**
LSTM	0.855	0.684	0.618	0.621***
Sample Period: 2017/12/01 – 2020/02/29				
Random Walk	0.363	–	–	0.364
AR Model	0.351	–	–	0.351
RanForest	0.363	0.380	0.361	0.366
Boosting	0.538	0.424	0.400	0.363*
SVM-RBF	1.502	0.366***	0.383***	0.448***
SVM-Poly	0.305	0.395	0.436	1.413
LASSO	0.363	0.363	0.364	0.371
Ridge	0.514	0.362	0.364	0.374**
Nnet2	0.883	0.356**	0.518**	0.364**
Nnet3	0.443	0.344	0.365	0.364
LSTM	0.460	0.464	0.335	0.366*
Sample Period: 2020/02/29 – 2023/12/01				
Random Walk	0.768	–	–	0.768
AR Model	0.782	–	–	0.782
RanForest	0.812	0.784	0.753	0.794
Boosting	0.822	0.821	0.789	0.770
SVM-RBF	1.977	0.773***	0.771***	0.861***
SVM-Poly	2.881	1.037***	1.070***	1.733***
LASSO	0.768	0.768	0.769	0.772
Ridge	1.705	0.777**	0.769**	0.774***
Nnet2	1.690	0.863*	1.130	0.769***
Nnet3	1.074	0.806	0.814	0.769**
LSTM	1.091	0.815	0.788	0.774***

Notes: All reported MSFE entries are actual MSFEs multiplied by 10^{-2} . See notes to Table 1 for further details.

Table 4: Model Confidence Set p -Values*JPM Monthly Returns*

Learner Model	Mixed-Frequency Method			
	Monthly	HPE	GHPA	DMHTA
Sample Period: 2017/12/01 – 2023/12/01				
Random Walk	0.9516	–	–	1.0000
AR Model	0.3366	–	–	1.0000
RanForest	0.7872	0.6882	1.0000	0.7872
Boosting	0.2922	0.5980	1.0000	0.9616
SVM-RBF	0.0000	1.0000	0.9354	0.5558
SVM-Poly	0.0704	1.0000	0.9864	0.0012
LASSO	0.9172	1.0000	0.9172	0.5352
Ridge	0.0638	0.8670	1.0000	0.8670
Nnet2	0.0182	0.6190	0.0672	1.0000
Nnet3	0.2984	0.8316	0.8944	1.0000
LSTM	0.0506	0.4642	1.0000	0.8162
Sample Period: 2017/12/01 – 2020/02/29				
Random Walk	0.9374	–	–	1.0000
AR Model	0.9218	–	–	1.0000
RanForest	0.8728	0.9958	1.0000	0.8728
Boosting	0.0662	0.9816	0.9816	1.0000
SVM-RBF	0.0000	0.4060	1.0000	0.2826
SVM-Poly	1.0000	0.8894	0.8894	0.0000
LASSO	0.8490	0.8574	1.0000	0.5520
Ridge	0.0386	1.0000	0.8538	0.7770
Nnet2	0.0428	1.0000	0.7016	0.7016
Nnet3	0.4062	1.0000	0.5224	0.5224
LSTM	0.4312	0.4312	1.0000	0.5424
Sample Period: 2020/02/29 – 2023/12/01				
Random Walk	0.9710	–	–	1.0000
AR Model	0.2906	–	–	1.0000
RanForest	0.8978	0.6914	1.0000	0.8978
Boosting	0.6866	0.6750	1.0000	0.9340
SVM-RBF	0.0000	1.0000	0.8330	0.8330
SVM-Poly	0.0330	0.9080	1.0000	0.0758
LASSO	0.9580	1.0000	0.9032	0.8558
Ridge	0.0336	0.8316	1.0000	0.9324
Nnet2	0.0486	0.5312	0.0618	1.0000
Nnet3	0.2670	0.5356	0.7930	1.0000
LSTM	0.0532	0.7096	1.0000	0.9568

Notes: Each cell reports the MCS p -value from the range statistic (T_R) of Hansen et al. (2011). See notes to Table 2 for further details.

Table 5: MSFEs of “Learner Models” Using Different Mixed-Frequency Methods

LLY Monthly Returns

Learner Model	Mixed-Frequency Method			
	Monthly	HPE	GHPA	DMHTA
Sample Period: 2017/12/01 – 2023/12/01				
Random Walk	0.609	–	–	0.611
AR Model	0.647	–	–	0.648
RanForest	0.581	0.576	0.558	0.605
Boosting	0.600	0.592	0.565	0.615
SVM-RBF	0.739	0.614	0.575	0.553**
SVM-Poly	0.688	0.962	0.630	1.113
LASSO	0.609	0.613	0.613	0.611
Ridge	0.581	0.551	0.610	0.567
Nnet2	0.856	0.568**	0.554**	0.604**
Nnet3	0.799	0.554*	0.560*	0.605*
LSTM	0.751	0.613	0.552	0.600***
Sample Period: 2017/12/01 – 2020/02/29				
Random Walk	0.339	–	–	0.340
AR Model	0.366	–	–	0.367
RanForest	0.336	0.335	0.338	0.340
Boosting	0.346	0.380	0.343	0.340
SVM-RBF	0.424	0.341	0.337	0.333
SVM-Poly	0.319	0.338	0.367	0.746
LASSO	0.339	0.340	0.340	0.339
Ridge	0.332	0.330	0.340	0.357
Nnet2	0.713	0.318***	0.310**	0.336**
Nnet3	0.433	0.327	0.322	0.336
LSTM	0.445	0.333	0.321	0.332**
Sample Period: 2020/02/29 – 2023/12/01				
Random Walk	0.772	–	–	0.774
AR Model	0.816	–	–	0.817
RanForest	0.729	0.721	0.690	0.764
Boosting	0.753	0.720	0.697	0.780
SVM-RBF	0.928	0.778	0.718	0.685**
SVM-Poly	0.910	1.337	0.788	1.333
LASSO	0.772	0.776	0.776	0.774
Ridge	0.730	0.684	0.772	0.693
Nnet2	0.943	0.718	0.701	0.765
Nnet3	1.018	0.690*	0.702	0.766
LSTM	0.934	0.781	0.690	0.760***

Notes: All reported MSFE entries are actual MSFEs multiplied by 10^{-2} . See notes to Table 1 for further details.

Table 6: Model Confidence Set p -Values*LLY Monthly Returns*

Learner Model	Mixed-Frequency Method			
	Monthly	HPE	GHPA	DMHTA
Sample Period: 2017/12/01 – 2023/12/01				
Random Walk	1.0000	–	–	0.0020
AR Model	1.0000	–	–	0.0058
RanForest	0.7818	0.7010	1.0000	0.4358
Boosting	0.8202	0.6872	1.0000	0.8202
SVM-RBF	0.0176	0.0690	0.7426	1.0000
SVM-Poly	0.5236	0.0946	1.0000	0.0040
LASSO	1.0000	0.9852	0.9852	0.1270
Ridge	0.9184	0.9932	0.7974	1.0000
Nnet2	0.0146	0.9854	1.0000	0.8478
Nnet3	0.0740	1.0000	0.8532	0.7732
LSTM	0.0180	0.2838	1.0000	0.5506
Sample Period: 2017/12/01 – 2020/02/29				
Random Walk	1.0000	–	–	0.3570
AR Model	1.0000	–	–	0.0694
RanForest	0.9004	1.0000	0.6330	0.9004
Boosting	0.9786	1.0000	0.9786	0.9786
SVM-RBF	0.7008	0.7562	1.0000	0.9168
SVM-Poly	0.9120	1.0000	0.3750	0.0264
LASSO	0.9904	1.0000	1.0000	0.9904
Ridge	0.9988	1.0000	0.9988	0.9504
Nnet2	0.0382	1.0000	0.5808	0.7160
Nnet3	0.5262	0.9500	1.0000	0.9290
LSTM	0.1382	0.9668	1.0000	0.9668
Sample Period: 2020/02/29 – 2023/12/01				
Random Walk	1.0000	–	–	0.0064
AR Model	1.0000	–	–	0.0206
RanForest	0.8526	0.4898	1.0000	0.4542
Boosting	0.8426	0.5322	1.0000	0.8426
SVM-RBF	0.0242	0.0924	0.7042	1.0000
SVM-Poly	0.5040	0.0798	1.0000	0.0798
LASSO	1.0000	0.9558	0.9558	0.0932
Ridge	0.8008	0.8620	0.8008	1.0000
Nnet2	0.3412	0.8880	1.0000	0.8880
Nnet3	0.1898	1.0000	0.8784	0.8784
LSTM	0.0556	0.3138	1.0000	0.6374

Notes: Each cell reports the MCS p -value from the range statistic (T_R) of Hansen et al. (2011). See notes to Table 2 for further details.

Table 7: MSFEs of “Learner Models” Using Different Mixed-Frequency Methods

XOM Monthly Returns

Learner Model	Mixed-Frequency Method			
	Monthly	HPE	GHPA	DMHTA
Sample Period: 2017/12/01 – 2023/12/01				
Random Walk	0.883	–	–	0.883
AR Model	0.890	–	–	0.888
RanForest	0.911	0.887	0.903	0.903
Boosting	0.937	0.940	0.903	0.875*
SVM-RBF	0.943	0.884	1.017	0.886*
SVM-Poly	3.647	1.015***	0.946***	1.043***
LASSO	0.883	0.884	0.885	0.883
Ridge	1.087	0.934	0.889	0.919***
Nnet2	1.127	0.972	0.929	0.884**
Nnet3	0.937	0.917	0.918	0.884*
LSTM	0.887	0.914	0.901	0.881
Sample Period: 2017/12/01 – 2020/02/29				
Random Walk	0.469	–	–	0.470
AR Model	0.467	–	–	0.467
RanForest	0.458	0.453	0.447	0.467
Boosting	0.535	0.539	0.470	0.463*
SVM-RBF	0.447	0.467	0.468	0.463
SVM-Poly	0.385	0.455	0.449	0.804
LASSO	0.469	0.458	0.464	0.471
Ridge	0.471	0.456	0.462	0.455
Nnet2	0.404	0.436	0.437	0.476
Nnet3	0.501	0.450	0.449	0.474
LSTM	0.420	0.421	0.450	0.478
Sample Period: 2020/02/29 – 2023/12/01				
Random Walk	1.132	–	–	1.130
AR Model	1.144	–	–	1.141
RanForest	1.182	1.147	1.177	1.165
Boosting	1.179	1.180	1.163	1.122
SVM-RBF	1.240	1.134	1.346	1.140**
SVM-Poly	5.604	1.351***	1.244***	1.186***
LASSO	1.132	1.139	1.137	1.131
Ridge	1.458	1.221	1.145	1.198***
Nnet2	1.560	1.293	1.225	1.128***
Nnet3	1.199	1.198	1.199	1.130*
LSTM	1.168	1.210	1.172	1.123

Notes: All reported MSFE entries are actual MSFEs multiplied by 10^{-2} . See notes to Table 1 for further details.

Table 8: Model Confidence Set p -Values*XOM Monthly Returns*

Learner Model	Mixed-Frequency Method			
	Monthly	HPE	GHPA	DMHTA
Sample Period: 2017/12/01 – 2023/12/01				
Random Walk	0.6112	–	–	1.0000
AR Model	0.3620	–	–	1.0000
RanForest	0.7616	1.0000	0.2540	0.9154
Boosting	0.1716	0.5066	0.8510	1.0000
SVM-RBF	0.2910	1.0000	0.1746	0.9808
SVM-Poly	0.0344	0.4198	1.0000	0.6356
LASSO	1.0000	1.0000	1.0000	0.9990
Ridge	0.1896	0.2360	1.0000	0.8204
Nnet2	0.1420	0.0194	0.8640	1.0000
Nnet3	0.2120	0.5348	0.8362	1.0000
LSTM	0.9910	1.0000	0.8382	0.9910
Sample Period: 2017/12/01 – 2020/02/29				
Random Walk	1.0000	–	–	0.1418
AR Model	0.9086	–	–	1.0000
RanForest	0.8588	1.0000	0.8444	0.8588
Boosting	0.0902	0.3000	1.0000	0.7870
SVM-RBF	0.8164	0.1478	1.0000	0.8164
SVM-Poly	1.0000	0.8188	0.5584	0.0016
LASSO	0.9100	0.9100	1.0000	0.4348
Ridge	0.7590	1.0000	0.2834	0.7590
Nnet2	0.8770	0.4866	1.0000	0.3392
Nnet3	0.6090	0.1014	1.0000	0.6090
LSTM	0.8892	1.0000	0.8892	0.7686
Sample Period: 2020/02/29 – 2023/12/01				
Random Walk	0.2228	–	–	1.0000
AR Model	0.3214	–	–	1.0000
RanForest	0.3860	0.9788	0.3216	1.0000
Boosting	0.3468	0.9540	0.9540	1.0000
SVM-RBF	0.1008	0.9484	0.0520	1.0000
SVM-Poly	0.0020	0.3334	0.8188	1.0000
LASSO	0.9724	0.9920	0.9920	1.0000
Ridge	0.1056	0.1056	1.0000	0.9018
Nnet2	0.0210	0.0094	0.7030	1.0000
Nnet3	0.2892	0.8324	0.8324	1.0000
LSTM	0.9022	0.9022	0.9022	1.0000

Notes: Each cell reports the MCS p -value from the range statistic (T_R) of Hansen et al. (2011). See notes to Table 2 for further details.

References

Hansen, P. R., Lunde, A., and Nason, J. M. (2011). The Model Confidence Set. *Econometrica*, 79(2), 453–497.